- The ***rotator*** $\mathbf{Q}$ that rotates vectors $\mathbf{u}$ in $\Re^2$ counterclockwise through an angle θ, as shown in Figure 4.7.1, is a linear operator on $\Re^2$ because the "action" of $\mathbf{Q}$ on $\mathbf{u}$ can be described by matrix multiplication in the sense that the coordinates of the rotated vector $\mathbf{Q}(\mathbf{u})$ are given by

$$\mathbf{Q}(\mathbf{u}) = \begin{pmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

- The ***projector*** $\mathbf{P}$ that maps each point $\mathbf{v} = (x, y, z) \in \Re^3$ to its orthogonal projection $(x, y, 0)$ in the xy-plane, as depicted in Figure 4.7.2, is a linear operator on $\Re^3$ because if $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$, then

$$\mathbf{P}(\alpha\mathbf{u} + \mathbf{v}) = (\alpha u_1 + v_1, \alpha u_2 + v_2, 0) = \alpha(u_1, u_2, 0) + (v_1, v_2, 0) = \alpha\mathbf{P}(\mathbf{u}) + \mathbf{P}(\mathbf{v}).$$

- The ***reflector*** $\mathbf{R}$ that maps each vector $\mathbf{v} = (x, y, z) \in \Re^3$ to its reflection $\mathbf{R}(\mathbf{v}) = (x, y, -z)$ about the xy-plane, as shown in Figure 4.7.3, is a linear operator on $\Re^3$.

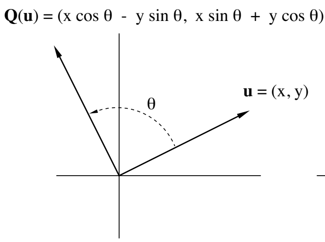

Figure 4.7.1

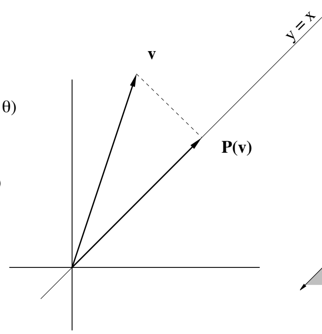

Figure 4.7.2

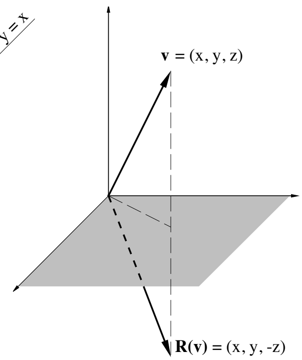

Figure 4.7.3

- Just as the rotator $\mathbf{Q}$ is represented by a matrix $[\mathbf{Q}] = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, the projector $\mathbf{P}$ and the reflector $\mathbf{R}$ can be represented by matrices

$$[\mathbf{P}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad [\mathbf{R}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

in the sense that the "action" of $\mathbf{P}$ and $\mathbf{R}$ on $\mathbf{v} = (x, y, z)$ can be accomplished with matrix multiplication using $[\mathbf{P}]$ and $[\mathbf{R}]$ by writing

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ -z \end{pmatrix}.$$