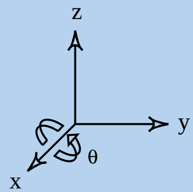
Rotations in $\mathbf{R}^3$

A vector $\mathbf{u} \in \Re^3$ can be rotated counterclockwise through an angle θ around a coordinate axis by means of a multiplication $\mathbf{P}_\star\mathbf{u}$ in which $\mathbf{P}_\star$ is an appropriate orthogonal matrix as described below.

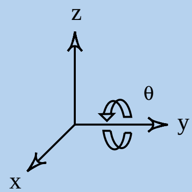
Rotation around the x-Axis

$$\mathbf{P}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$



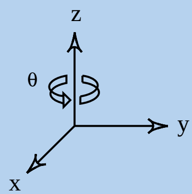
Rotation around the y-Axis

$$\mathbf{P}_y = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$



Rotation around the z-Axis

$$\mathbf{P}_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Note: The minus sign appears above the diagonal in $\mathbf{P}_x$ and $\mathbf{P}_z$, but below the diagonal in $\mathbf{P}_y$. This is not a mistake—it's due to the orientation of the positive x-axis with respect to the yz-plane.

Example 5.6.4

3-D Rotational Coordinates. Suppose that three counterclockwise rotations are performed on the three-dimensional solid shown in Figure 5.6.5. First rotate the solid in View (a) $90°$ around the x-axis to obtain the orientation shown in View (b). Then rotate View (b) $45°$ around the y-axis to produce View (c) and, finally, rotate View (c) $60°$ around the z-axis to end up with View (d). You can follow the process by watching how the notch, the vertex $\mathbf{v}$, and the lighter shaded face move.