symmetric about the point $n/2$ in the frequency domain, and the information in just the first (or second) half of the frequency domain completely characterizes the original waveform—this is why only 512/2=256 points are plotted in the graphs shown in Figure 5.8.4. In other words, if

$$\mathbf{y} = \frac{2}{n}\mathbf{F}_n\mathbf{x} = \sum_k \alpha_k\left(\mathbf{e}_{f_k} + \mathbf{e}_{n-f_k}\right) + \mathrm{i}\sum_k \beta_k\left(-\mathbf{e}_{f_k} + \mathbf{e}_{n-f_k}\right), \tag{5.8.8}$$

then the information in

$$\mathbf{y}_{n/2} = \sum_k \alpha_k \mathbf{e}_{f_k} - \mathrm{i}\sum_k \beta_k \mathbf{e}_{f_k} \qquad \text{(the first half of } \mathbf{y}\text{)}$$

is enough to reconstruct the original waveform. For example, the equation of the waveform shown in Figure 5.8.7 is

$$x(\tau) = 3\cos 2\pi\tau + 5\sin 2\pi\tau, \tag{5.8.9}$$

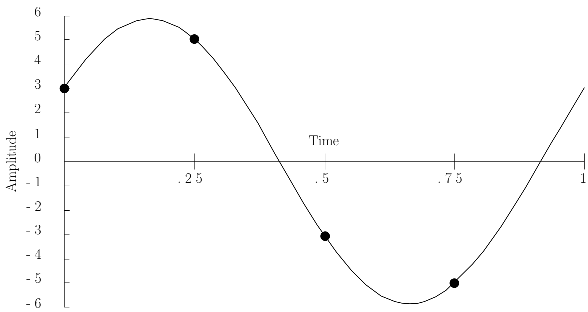

FIGURE 5.8.7

and it is completely determined by the four values in

$$\mathbf{x} = \begin{pmatrix} x(0) \\ x(1/4) \\ x(1/2) \\ x(3/4) \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -3 \\ -5 \end{pmatrix}.$$

To capture equation (5.8.9) from these four values, compute the vector $\mathbf{y}$ defined by (5.8.8) to be

$$\mathbf{y} = \frac{2}{4}\mathbf{F}_4\mathbf{x} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -\mathrm{i} & -1 & \mathrm{i} \\ 1 & -1 & 1 & -1 \\ 1 & \mathrm{i} & -1 & -\mathrm{i} \end{pmatrix}\begin{pmatrix} 3 \\ 5 \\ -3 \\ -5 \end{pmatrix} = \begin{pmatrix} 0 \\ 3-5\mathrm{i} \\ 0 \\ 3+5\mathrm{i} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 3 \\ 0 \\ 3 \end{pmatrix} + \mathrm{i}\begin{pmatrix} 0 \\ -5 \\ 0 \\ 5 \end{pmatrix} = 3(\mathbf{e}_1 + \mathbf{e}_3) + 5\mathrm{i}(-\mathbf{e}_1 + \mathbf{e}_3).$$