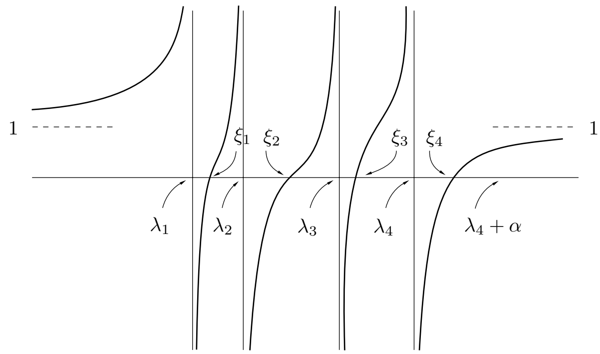


FIGURE 7.1.4

(b) Verify that $(\mathbf{D}-\xi_i\mathbf{I})^{-1}\mathbf{v}$ is an eigenvector for $\mathbf{D}+\alpha\mathbf{v}\mathbf{v}^T$ that is associated with the eigenvalue $\xi_i$.

**7.1.23. Newton's Identities.** Let $\lambda_1,\ldots,\lambda_n$ be the roots of the polynomial $p(\lambda)=\lambda^n+c_1\lambda^{n-1}+c_2\lambda^{n-2}+\cdots+c_n$, and let $\tau_k=\lambda_1^k+\lambda_2^k+\cdots+\lambda_n^k$. Newton's identities say $c_k=-(\tau_1c_{k-1}+\tau_2c_{k-2}+\cdots+\tau_{k-1}c_1+\tau_k)/k$. Derive these identities by executing the following steps:

(a) Show $p'(\lambda)=p(\lambda)\sum_{i=1}^n(\lambda-\lambda_i)^{-1}$ (logarithmic differentiation).

(b) Use the geometric series expansion for $(\lambda-\lambda_i)^{-1}$ to show that for $|\lambda|>\max_i|\lambda_i|$,

$$\sum_{i=1}^n\frac{1}{(\lambda-\lambda_i)}=\frac{n}{\lambda}+\frac{\tau_1}{\lambda^2}+\frac{\tau_2}{\lambda^3}+\cdots.$$

(c) Combine these two results, and equate like powers of $\lambda$.

**7.1.24. Leverrier–Souriau–Frame Algorithm.**[69] Let the characteristic equation for $\mathbf{A}$ be given by $\lambda^n+c_1\lambda^{n-1}+c_2\lambda^{n-2}+\cdots+c_n=0$, and define a sequence by taking $\mathbf{B}_0=\mathbf{I}$ and

$$\mathbf{B}_k=-\frac{trace\,(\mathbf{AB}_{k-1})}{k}\mathbf{I}+\mathbf{AB}_{k-1}\quad\text{for}\quad k=1,2,\ldots,n.$$

Prove that for each $k$,

$$c_k=-\frac{trace\,(\mathbf{AB}_{k-1})}{k}.$$

**Hint:** Use Newton's identities, and recall Exercise 7.1.10(a).

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[69] This algorithm has been rediscovered and modified several times. In 1840, the Frenchman U. J. J. Leverrier provided the basic connection with Newton's identities. J. M. Souriau, also from France, and J. S. Frame, from Michigan State University, independently modified the algorithm to its present form—Souriau's formulation was published in France in 1948, and Frame's method appeared in the United States in 1949. Paul Horst (USA, 1935) along with Faddeev and Sominskii (USSR, 1949) are also credited with rediscovering the technique. Although the algorithm is intriguingly beautiful, it is not practical for floating-point computations.