

# Contents

<b>Preface</b> . . . . .	ix
<b>1. Linear Equations</b> . . . . .	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Gaussian Elimination and Matrices . . . . .	3
1.3 Gauss–Jordan Method . . . . .	15
1.4 Two-Point Boundary Value Problems . . . . .	18
1.5 Making Gaussian Elimination Work . . . . .	21
1.6 Ill-Conditioned Systems . . . . .	33
<b>2. Rectangular Systems and Echelon Forms</b> . . . . .	<b>41</b>
2.1 Row Echelon Form and Rank . . . . .	41
2.2 Reduced Row Echelon Form . . . . .	47
2.3 Consistency of Linear Systems . . . . .	53
2.4 Homogeneous Systems . . . . .	57
2.5 Nonhomogeneous Systems . . . . .	64
2.6 Electrical Circuits . . . . .	73
<b>3. Matrix Algebra</b> . . . . .	<b>79</b>
3.1 From Ancient China to Arthur Cayley . . . . .	79
3.2 Addition and Transposition . . . . .	81
3.3 Linearity . . . . .	89
3.4 Why Do It This Way . . . . .	93
3.5 Matrix Multiplication . . . . .	95
3.6 Properties of Matrix Multiplication . . . . .	105
3.7 Matrix Inversion . . . . .	115
3.8 Inverses of Sums and Sensitivity . . . . .	124
3.9 Elementary Matrices and Equivalence . . . . .	131
3.10 The LU Factorization . . . . .	141
<b>4. Vector Spaces</b> . . . . .	<b>159</b>
4.1 Spaces and Subspaces . . . . .	159
4.2 Four Fundamental Subspaces . . . . .	169
4.3 Linear Independence . . . . .	181
4.4 Basis and Dimension . . . . .	194

4.5	More about Rank . . . . .	210
4.6	Classical Least Squares . . . . .	223
4.7	Linear Transformations . . . . .	238
4.8	Change of Basis and Similarity . . . . .	251
4.9	Invariant Subspaces . . . . .	259
<b>5.</b>	<b>Norms, Inner Products, and Orthogonality . . . . .</b>	<b>269</b>
5.1	Vector Norms . . . . .	269
5.2	Matrix Norms . . . . .	279
5.3	Inner-Product Spaces . . . . .	286
5.4	Orthogonal Vectors . . . . .	294
5.5	Gram–Schmidt Procedure . . . . .	307
5.6	Unitary and Orthogonal Matrices . . . . .	320
5.7	Orthogonal Reduction . . . . .	341
5.8	Discrete Fourier Transform . . . . .	356
5.9	Complementary Subspaces . . . . .	383
5.10	Range-Nullspace Decomposition . . . . .	394
5.11	Orthogonal Decomposition . . . . .	403
5.12	Singular Value Decomposition . . . . .	411
5.13	Orthogonal Projection . . . . .	429
5.14	Why Least Squares? . . . . .	446
5.15	Angles between Subspaces . . . . .	450
<b>6.</b>	<b>Determinants . . . . .</b>	<b>459</b>
6.1	Determinants . . . . .	459
6.2	Additional Properties of Determinants . . . . .	475
<b>7.</b>	<b>Eigenvalues and Eigenvectors . . . . .</b>	<b>489</b>
7.1	Elementary Properties of Eigensystems . . . . .	489
7.2	Diagonalization by Similarity Transformations . . . . .	505
7.3	Functions of Diagonalizable Matrices . . . . .	525
7.4	Systems of Differential Equations . . . . .	541
7.5	Normal Matrices . . . . .	547
7.6	Positive Definite Matrices . . . . .	558
7.7	Nilpotent Matrices and Jordan Structure . . . . .	574
7.8	Jordan Form . . . . .	587
7.9	Functions of Nondiagonalizable Matrices . . . . .	599

7.10	Difference Equations, Limits, and Summability . . .	616
7.11	Minimum Polynomials and Krylov Methods . . .	642
<b>8.</b>	<b>Perron–Frobenius Theory . . . . .</b>	<b>661</b>
8.1	Introduction . . . . .	661
8.2	Positive Matrices . . . . .	663
8.3	Nonnegative Matrices . . . . .	670
8.4	Stochastic Matrices and Markov Chains . . . . .	687
	<b>Index . . . . .</b>	<b>705</b>

*You are today where your knowledge brought you;  
you will be tomorrow where your knowledge takes you.*  
— *Anonymous*